Name:	
Student ID:	

Example Examination

Phys331: Electromagnetic Theory I

2025/09/29

Please carefully read below before proceeding!

I acknowledge by taking this examination that I am aware of all academic honesty conducts that govern this course and how they also apply for this examination. I therefore accept that I will not engage in any form of academic dishonesty including but not limited to cheating or plagiarism. I waive any right to a future claim as to have not been informed in these matters because I have read the syllabus along with the academic integrity information presented therein.

I also understand and agree with the following conditions:

(1) all calculations are to be conducted in the notations and conventions of the formulae sheets provided during the exam unless explicitly stated otherwise in the question;

(2) I take *full responsibility* for any ambiguity in my selections in “multiple choice questions”;

(3) incorrect selections will receive $-1/7$ of the question’s points;

(4) I am expected to provide *step-by-step explanation of how I solved the question* and am expected to do so *only within the answer boxes* provided with the questions: the explanation is supposed to be succinct, well-articulated, and correct both scientifically and mathematically;

(5) no partial credit is awarded for the explanations provided in the answer boxes;

(6) some questions of some students will be randomly selected for inspection: *a question (if selected for inspection) might be awarded negative points* if its explanation is incorrect or insufficient to get the correct answer, even if the correct option is selected;

(7) any page which does not contain *both my name and student id* will not be graded;

(8) any extra sheet that I may use are for my own calculations and will not be graded.

Signature: ______________________________

This exam has a total of 3 questions, some of which may be for bonus points. You can obtain a maximum grade of 105+0 from this examination.

Question:	1	2	3	Total
Points:	7	21	77	105

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Question: 1: Concept of Units (A simple case study) . . . . *(7 points)*

In cosmology, the kinematics are handled using a different unit-system than SI; indeed, although *meter* and *second* are the standard units for most of the engineering and applied physics, they are too small for cosmological scales so instead we might prefer *lightyear* and *gigayear* which we can take to be $\mathrm{ly} = 10^{16}\,\mathrm{m}\ \ \mathrm{Gyr} = 3\times 10^{16}\,\mathrm{s}$. Then, if absolute value of the angular momentum per units mass for the galaxy Andromeda is approximated as $|\boldsymbol{j}| = 2\times 10^{26}\,\mathrm{m}^2\ \mathrm{s}^{-1}$ in the SI units, which option below would be the correct expression for it?

☐ $|\boldsymbol{j}| = 6^{-1}\times 10^{16}\,\mathrm{ly}^2\,\mathrm{Gyr}^{-1}$ ☐ $|\boldsymbol{j}| = 6\times 10^{16}\,\mathrm{ly}^2\,\mathrm{Gyr}^{-1}$ ■ $|\boldsymbol{j}| = 6\times 10^{10}\,\mathrm{ly}^2\,\mathrm{Gyr}^{-1}$ ☐ $|\boldsymbol{j}| = \frac{2}{3}\times 10^{10}\,\mathrm{ly}^2\,\mathrm{Gyr}^{-1}$

☐ $|\boldsymbol{j}| = \frac{2}{3}\times 10^{-10}\,\mathrm{ly}^2\,\mathrm{Gyr}^{-1}$ ☐ $|\boldsymbol{j}| = 6\times 10^{-10}\,\mathrm{ly}^2\,\mathrm{Gyr}^{-1}$ ☐ $|\boldsymbol{j}| = 6\times 10^{-16}\,\mathrm{ly}^2\,\mathrm{Gyr}^{-1}$ ☐ $|\boldsymbol{j}| = 6^{-1}\times 10^{-16}\,\mathrm{ly}^2\,\mathrm{Gyr}^{-1}$

Please provide below the step-by-step explanation of how you obtained your result(s) for question 1:

We can invert given relation between lightyear and meter (and similarly between gigayear and second) and insert these into the expression for $|\boldsymbol{j}|$: $|\boldsymbol{j}| = 2\times 10^{26}\left(10^{-16}\mathrm{ly}\right)^2\left(3^{-1}\times 10^{-16}\mathrm{Gyr}\right)^{-1}$.

Question: 2: Concept of Units (A complicated case study) . . . . *(21 points)*

On his 1881 paper *"On the physical units of nature"*, G. Johnstone Stoney argues the utility of choosing fundamental units in terms of constants of nature, hence creating a unit system which is named after him. In this so-called *Stoney natural units*, we trade the SI units A, m, kg, s for the constants of nature c, e, G, ϵ_0 which we will take to be defined as

$$\begin{aligned}
\text{(speed of light) } c &= 3\times 10^{8}\,\mathrm{m}\ \mathrm{s}^{-2}\\
\text{(charge of proton) } e &= 2\times 10^{-19}\,\mathrm{A}\ \mathrm{s}\\
\text{(gravitational constant) } G &= 7\times 10^{-11}\,\mathrm{m}^3\ \mathrm{kg}^{-1}\ \mathrm{s}^{-2}\\
\text{(vacuum permittivity) } \epsilon_0 &= 9\times 10^{-12}\,\mathrm{A}^2\ \mathrm{m}^{-3}\ \mathrm{kg}^{-1}\ \mathrm{s}^{4}
\end{aligned} \tag{1}$$

in this question.

Answer the questions below based on these definitions, along with the facts

$$\sqrt{630}\approx 25 \quad , \quad \begin{pmatrix} 0 & 1 & 0 & -2\\ 1 & 0 & 0 & 1\\ 0 & 3 & -1 & -2\\ 2 & -3 & -1 & 4 \end{pmatrix}^{-1} = \frac{1}{8}\begin{pmatrix} 6 & 6 & -1 & 1\\ -4 & 4 & 2 & -2\\ 0 & 8 & -4 & -4\\ -6 & 2 & 1 & -1 \end{pmatrix} \tag{2}$$

(a) **(7 points)** Consider Alice, whose mass is measured to be "$80\,\mathrm{A}^0\ \mathrm{kg}^1\ \mathrm{s}^0\ \mathrm{m}^0$" in SI units. In Stoney natural units, her mass would be "$X c^{a_1} e^{a_2} G^{a_3} \epsilon_0^{a_4}$" for some exponents a_i. What is X?

☐ 10^3 ☐ 10^4 ☐ 10^5 ☐ 10^6

☐ 10^7 ☐ 10^8 ☐ 10^9 ■ 10^{10}

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(b) **(7 points)** Consider Alice again: which of below would be her mass in Stoney natural units?

■ $X c^0 e^1 G^{-1/2} \epsilon_0^{-1/2}$ ☐ $X c^0 e^1 G^{1/2} \epsilon_0^{-1/2}$ ☐ $X c^0 e^1 G^{-1/2} \epsilon_0^{1/2}$ ☐ $X c^0 e^1 G^{1/2} \epsilon_0^{1/2}$
☐ $X c^0 e^{-1/2} G^1 \epsilon_0^{-1/2}$ ☐ $X c^0 e^{-1/2} G^1 \epsilon_0^{1/2}$ ☐ $X c^0 e^{1/2} G^1 \epsilon_0^{-1/2}$ ☐ $X c^0 e^{1/2} G^1 \epsilon_0^{1/2}$

(c) **(7 points)** Now assume that Alice has done some work, say 10 Joules in SI units. What would be this in Stoney natural units for some $Y \in \mathbb{R}$?

☐ $Y c^{1/2} e^{3/4} G^{1/4} \epsilon_0^{-3/4}$ ☐ $Y c^{1/2} e^{3/2} G^{1/4} \epsilon_0^{-3/4}$ ☐ $Y c^{1/2} e^{3/4} G^{-1/4} \epsilon_0^{-3/4}$ ■ $Y c^{1/2} e^{3/2} G^{-1/4} \epsilon_0^{-3/4}$
☐ $Y c^{-1/2} e^{3/4} G^{1/4} \epsilon_0^{-3/4}$ ☐ $Y c^{-1/2} e^{3/2} G^{1/4} \epsilon_0^{-3/4}$ ☐ $Y c^{-1/2} e^{3/4} G^{-1/4} \epsilon_0^{-3/4}$ ☐ $Y c^{-1/2} e^{3/2} G^{-1/4} \epsilon_0^{-3/4}$

Please provide below the step-by-step explanation of how you obtained your result(s) for question 1:

We can solve all three parts together if we determine how a generic derived unit in SI, i.e. $m_0\ \mathrm{A}^{m_1}\ \mathrm{m}^{m_2}\ \mathrm{kg}^{m_3}\ \mathrm{s}^{m_4}$, is related to a generic derived unit in Stoney natural units, i.e. $a_0 c^{a_1} e^{a_2} G^{a_3} \epsilon_0^{a_4}$. We can do this as follows:

1. Write each SI unit in terms of Stoney units, with coefficients to be determined, for instance

$$\mathrm{m} = \beta_0 c^{\beta_1} e^{\beta_2} G^{\beta_3} \epsilon_0^{\beta_4} \tag{3}$$

   We will have four such equations, hence 16 total exponents to be determined.
2. Insert these equations in the given equations for fundamental constants, i.e.

$$c = 3\times 10^8\ \mathrm{m\ s}^{-2} = \left(3\times 10^8 \beta_0 (\#)^{-2}\right) c^{\beta_1 - 2\#} e^{\beta_2 - 2\#} G^{\beta_3 - 2\#} \epsilon_0^{\beta_4 - 2\#} \tag{4}$$

   where exponents are linear combinations of undetermined coefficients. Matching the exponents give us four equation for each fundamental constants, hence 16 linear equations.
3. We have 16 linear equations for 16 undetermined coefficients; the rest is simply linear algebra: rewrite the equations as a matrix equation, invert the matrix to find the coefficients. The question already provides us with a matrix inverse, so clearly use it!
4. With the exponents determined, we can solve for the factors in the front (such as β_0).
5. With each SI unit expressible in Stoney units, we can now solve the parts straightforwardly by simply using relevant transformation.

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Question: Some other questions .... *(77 points)*

⋮

In the actual exam, there would be four to six questions, each with possibly two to three parts, totaling to 105 for the total score. It may be 6 total parts of 7 points each and 6 total parts of 10.5 points each ($6 \times 7 + 6 \times 10.5 = 105$), 3 total parts of 7 points each and 8 total parts of 10.5 points each ($3 \times 7 + 8 \times 10.5 = 105$), or any other appropriate point distribution.

As there will be four examinations in total with the same format (3 midterms and 1 final), your final letter grade will be based on 52 ± 8 gradable question parts in total.

« « « Congratulations, you have made it to the end! » » »