

Suggested Problems

FOR THE COURSES PHYS209 & PHYS210

Please Read

This document contains suggested problems for the courses Mathematical Methods in Physics I-II (Phys209, Phys210). The list is *not necessarily* comprehensive; however, it would give the student an idea of how well they are doing in this course: they are expected to be able to solve all these questions!

Although students are expected to be able to solve *all* questions in this list, *I as the instructor do not suggest them to sit down and solve all questions!* Depending on how well they are doing, on how much time they can spare for this course, and on how useful they find solving these questions are for them, they should themselves decide to choose which subset of these questions they would like to solve!

Books

All suggested problems are from the books specified in the syllabus, namely

(1) *Elementary Differential Equations and Boundary Value Problems* by Boyce and DiPrima (10th edition: ISBN 978-0-470-45831-0).

(2) *Advanced Calculus for Applications* by Hildebrand (1962 edition: ISBN 978-0-130-11163-0)

(3) *Mathematical Methods in the Physical Sciences* by Boas (3rd edition: ISBN 978-0-471-19826-0)

Phys209

1 Linear equations with constant coefficients

Characteristic equation, repeated roots, initial & boundary value problems:

Section 3.3, Problems 1-22 (page 164 [1])

Section 4.2, Problems 11-24, 37, 39 (page 235-236 [1])

Laplace transform:

Section 6.1, Problems 1-23, 30, 31 (page 315-316 [1])

Section 6.2, Problems 5-20 (page 325 [1])

Convolution and particular solution:

Section 6.6, Problems 4-11, 13-15, 17-20 (page 355 [1])

2 Linear equations with functional coefficients

2.1 Homogeneous Solution

Reparametrization, Euler equations:

Section 3.3, Problems 34-46 (page 165-166 [1])

Section 3.4, Problems 40-45 (page 175 [1])

Equations with dependent variable missing:

Section 2.9, Problems 36, 37, 40 (page 135 [1])

Exact equations:

Section 3.2, Problems 41-45 (page 157 [1])

Reduction of order:

Section 3.4, Problems 23-30 (page 174 [1])

Wronskian:

Section 3.3, Problem 27 (page 165 [1])

Section 4.1, Problems 11-16 (page 226 [1])

Series expansion around an ordinary point:

Section 5.1, Problems 9-28 (page 253-254 [1])

Section 5.2, Problems 1-14 (page 263-264 [1])

Section 5.3, Problems 1,22,27 (page 269-271 [1])

Series expansion around a singular point:

Section 5.4, Problems 1-12, 17-34 (page 280 [1])

Section 5.5, Problems 1-10 (page 286 [1])

Section 5.6, Problems 1-12 (page 294 [1])

2.2 Particular Solution

Method of undetermined coefficients:

Section 3.5, Problems 1-20 (page 184 [1])

Section 3.6, Problems 1-4 (page 190 [1])

Method of variation of parameters:

Section 3.6, Problems 5-18, 28-32 (page 190-192 [1])

3 Systems of first order linear differential equations

Solving general linear ordinary first order differential equations:

Section 2.1, Problems 13-20 (page 40 [1])

Conversion between arbitrary linear ordinary differential equations and systems of first order linear differential equations:

Section 7.1, Problems 1-12 (page 363-364 [1])

Basics of linear algebra:

Section 7.2, Problems 1-26 (page 376-378 [1])

Section 7.3, Problems 1-11, 16-25 (page 388-389 [1])

Analysis of coupled systems via matrix differential equations:

Section 7.5, Problems 9-18, 32-33 (page 405-407 [1])

Section 7.6, Problems 7-10, 25-26, 29 (page 417-420 [1])

Section 7.9, Problems 1-12 (page 447-448 [1])

4 Partial differential equations

Method of separation of variables:

Section 10.5, Problems 1-12, 20-23 (page 630-631 [1])

Wave equation:

Section 10.7, Problems 12-14, 16-18 (page 653-654 [1])

Phys210

5 Functions of a complex variable

Fundamentals:

Chapter 10, Problems 1-13, 19 (page 585-587 [2])

Complex integration:

Chapter 10, Problems 25-31 (page 588-589 [2])

Analyticity, singularities & residues:

Chapter 10, Problems 42-44, 50, 52, 57-67, 70, 72 (page 590-597 [2])

Inner product of complex functions:

Section 3.14, Problems 11-14 (page 184 [3])

Fourier transform:

Section 7.12, Problems 3-12, 21 (page 384-385 [3])

Fourier series:

Section 7.7, Problems 1-13 (page 360 [3])

6 Fundamentals of Vector Spaces

Ingredients for analysis of generic vector fields and their tensor generalization:

In class, we covered various topics: groups, rings, (skew-)fields, linear spaces, linear and lie algebras, structures on linear spaces (inner product & norm), dual vector spaces, basis expansion of vectors and covectors, relation among arrays of scalars and (co)vectors, Einstein's summation convention, tensors of arbitrary rank (n, m) , matrix representation of (co)vectors and rank-2 (meaning $n + m = 2$) tensors, and (co)vector & tensor fields.

Almost all of these concepts are absent in the books, at least to the degree that we have covered in the classes. Therefore, *you are not responsible to know any of these concepts in the examinations unless they are explicitly provided in the exams themselves.*

There is a dedicated chapter in [3] (chapter 10) to the tensor analysis; however, the chapter is explicitly designed towards *three dimensional Euclidean vector spaces*. As stated in the class repeatedly, three dimensional vectors are quite special in many regards (such as existence of the cross product) and Euclidean spaces are even more so (such as equivalence between covariant and contravariant vectors); therefore, that chapter would look rather different compared to how we discussed these topics in the class. You are encouraged to read/skim that chapter for the sake of your own background, however please do not expect much familiarity.

Basics of three dimensional Euclidean vector fields:

Chapter 6, Problems 1-3, 6, 8-12, 14 (page 314-316 [2])

Curvilinear Coordinate Transformations:

Section 5.4, Problems 5, 11, 14, 19, 20, 27 (page 268-270 [3])

Chapter 6, Problems 31 (page 319 [2])

Chapter 7, Problems 18 (page 370 [2])

7 Differentiation of Vector Fields

Gradient and directional derivative:

Section 6.6, Problems 1-15 (page 294-295 [3])

Divergence, Curl, Laplacian:

Section 6.7, Problems 1-20 (page 298-299 [3])

Derivatives with respect to external parameters:

Section 6.4, Problems 1-10 (page 289 [3])

8 Integration of Vector Fields

Line integrals:

Section 6.8, Problems 1-20 (page 306-308 [3])

Integral theorems for vector fields (applications of generalized Stoke's theorem):

Section 6.9, Problems 2-12 (page 313-314 [3])

Section 6.10, Problems 1-11 (page 322-323 [3])

Section 6.11, Problems 1-7,9-15 (page 334-335 [3])